Name of College — S. S. College, J-Bad 1

Deptt — Mathematics

Topic — Curve Tracing

Class — B.Sc I (Hons)

Time - 10.15 A.M to 11.A.M

11.00 A.M to 11.45 A.M

Date - 22-09-2020

Study material

Introduction

The Purpose of this Topic is to Know how to draw a graphs of curves whose equations are given. This we shall need in order to know the Area, Length, volume etc.

(1) Find if the Curve is symmetrical about any line.

(i) A curve is symmetrical about x-axis if the equation has only even powers of y

Ex:→ $Y^2 = 4ax$

(ii) A curve is symmetrical about y-axis if its equation only contains even power of x

Ex:→ $x^2 = 4ay$.

(iii) If the equation of curve contains only even power of x and y it is symmetrical about both axes

Ex:→ $x^2 + y^2 = a^2$

(iv) If on interchanging x and y the equation of the curve remains unchange

It is symmetrical about the line
$y = x$

Ex:→ $x^3 + y^3 = 3axy$
it is symmetrical about the line $y = x$

[Symmetrical about any lines means that on both sides of the line the curve has similar parts]

II → Find whether the curve passes through the origin

If on putting $x = 0$, $y = 0$ in the equation of curve, if the equation is satisfied the curve passing the origin.

Ex:→ $x^2 = 4ay$ passes through origin.

III Find the equation of tangent or tangents at origin.

for equate lowest degree term $= 0$ we get the equation of tangent at origin.

$x^2 = 4ay$

Here lowest degree term $4ay = 0$
$\Rightarrow y = 0$
$\Rightarrow$ x axis is the tangent at origin.

(IV) Find the point of intersection of the curve with axes. (3)

point of intersection of curve with x-axis

put $y=0$ in the equation of cur

point of intersection of curve with y-axis put $x=0$ in the equation of curve.

(V) Find the region of the curve.

for this, solve the equation for y. Find such values of x for which y bec imaginary

Ex:→ if the curve is

$$y^2(2a-x) = x^3$$

$$\Rightarrow y^2 = \frac{x^3}{2a-x}$$

if $x<0$, y is imaginary

and if $x>2a$ y is again imaginary

so No part of the curve is either in the left of the origin or in the right of ~~the origin~~. $x=2a$

See the behaviour of y (or x) for different values of x (or y) giving particular importance to those val for which y tends to ∞ or zero.

(vi) Find $\frac{dy}{dx}$ and the points where tangents are parallel to axes. (4)

(vii) Find the asymptote or asymptotes, if any.

Equating to zero, the co-efficient of highest power of y we get asymptote parallel to y-axis and equating to zero the co-efficient of highest power of x we get Asymptote parallel to x-axis

If the curve be of nth degree and the term containing y^n be absent then on equating the co-efficient of $y^{n-1} = 0$ given asymptotes parallel to y-axis.

If the equation of curve be of nth degree and co-efficient of $x^n \neq 0$

$\Rightarrow$ No asymptote is parallel to x-axis

Similarly if the co-efficient of $y^n \neq 0$

$\Rightarrow$ No asymptote is parallel to y-axis.

Tracing of Polar curve

(1) Symmetry

1) (a) Replace θ by $-\theta$.
if the equation of curve remains unchanged
the curve is symmetrical about initial line

(b) Replace r by $-r$, if the equation of
Curve remains unchanged
the Curve is symmetrical about pole.

(c) Replace θ by $\pi-\theta$, if the equation
of curve is unchanged,
the curve is symmetrical
about the line $\theta = \pi/2$
i.e (symmetrical about OX)

(d) Replace θ by $\pi/2-\theta$, if the equ
remains unchanged
the curve is symmetrical
the line $\theta = \pi/4$

(11) Whether the curve passes throu
pole —
If $r=0$ for some real value of θ.
the curve passes through the

If the curve passes through
pole, the value of θ for which
gives the equation of tangent
If $r=0$ for more than one value
the curve is said to have

(iii) Intersection of the curve with the initial line Ox and also with OY

for this

we know r by putting $\theta = 0$ and $\theta = \pi/2$ in the equation of the curve.

(iv) Region of the curve

To find the region in which no portion of the curve lies, we calculate those values of θ for which r is imaginary

(v) Asymptotes, if any

If the curve possesses never-ending branch, find its asymptote. This is sometimes best accomplished by transforming the equation to the Cartesian form.

(vi) Some more point → Some specific points on the curve should be plotted. for this we determine corresponding values of r for some special values of θ

for $\theta = 0, \pi/6, \pi/4, \pi/3, \pi/2$ -- ah plot them